

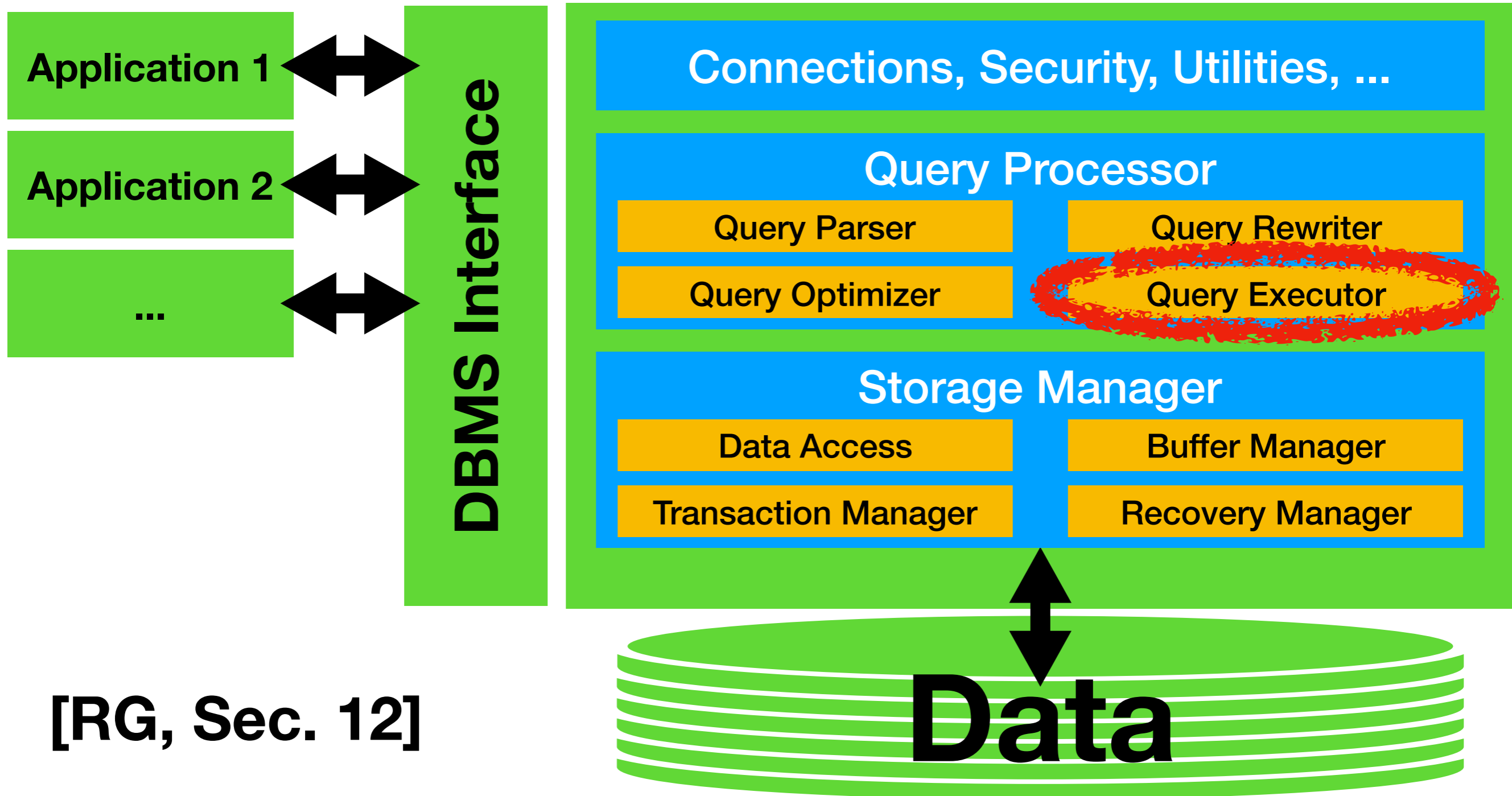
Hash Join, Sort-Merge Join

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Database Management Systems (DBMS)



[RG, Sec. 12]

Alternatives for Equality Joins?

Hash Join

- Want tuples with **same value** in join column
- Same value in join column implies **same hash** value
- Join **Phase 1**
 - **Partition data** by hash values in join columns
 - Make partitions small enough to **fit into memory**
- Join **Phase 2**
 - Join each partition pair (same hash value) **separately**

More Notations

- **Hash**(Tuple): Calculates hash function for tuple
- **Full**(P): Whether page P has no more space left
- **WriteAndClear**(P): Write P to disk and erase

Hash Join: Phase 1

$\bowtie E.Sid=S.Sid$

For ep in **Pages**(E):

LoadPage(ep)

For et in **Tuples**(ep):

Add et to $EB[\mathbf{Hash}(et)]$

If (**Full**($EB[\mathbf{Hash}(et)]$)):

WriteAndClear($EB[\mathbf{Hash}(et)]$)

Hash Join: Phase 1

$\bowtie E.Sid=S.Sid$

For ep in Pages(E):

LoadPage(ep) \longleftarrow *For each page in E*

For et in Tuples(ep):

Add et to EB[Hash(et)]

If (Full(EB[Hash(et))]):

WriteAndClear(EB[Hash(et))])

Hash Join: Phase 1

$\bowtie E.Sid=S.Sid$

For ep in Pages(E):

LoadPage(ep) \longleftarrow *For each page in E*

For et in Tuples(ep):

Add et to EB[Hash(et)]

If (Full(EB[Hash(et))]):

WriteAndClear(EB[Hash(et))) \longleftarrow *For each page in E*

Hash Join: Phase 1

$\bowtie E.Sid=S.Sid$

For ep in Pages(E):

LoadPage(ep) \longleftarrow *For each page in E*

For et in Tuples(ep):

Add et to EB[Hash(et)]

If (Full(EB[Hash(et))]):

WriteAndClear(EB[Hash(et))) \longleftarrow *For each page in E*

*Cost = pages in E * IO cost * 2*

Hash Join: Phase 1

$\bowtie E.Sid=S.Sid$

For sp in **Pages**(S):

LoadPage(sp) \longleftarrow *For each page in S*

For st in **Tuples**(sp):

Add st to $SB[\mathbf{Hash}(st)]$

If (**Full**($SB[\mathbf{Hash}(st)]$)):

WriteAndClear($SB[\mathbf{Hash}(st)]$) \longleftarrow *For each page in S*

*Cost = pages in S * IO cost * 2*

Hash Join: Phase 2

$\bowtie E.Sid=S.Sid$

For h in Hash Values:

LoadPages(EB[h])

For sp in **Pages**(SB[h]):

Load(sp)

For ep in **Pages**(EB[h]), st in sp, et in ep:

If (et.Sid=st.Sid):

Output(et \bowtie st)

Hash Join: Phase 2

$\bowtie E.Sid=S.Sid$

For h in Hash Values:

LoadPages(EB[h]) \longleftarrow *For each page in E*

For sp in **Pages**(SB[h]):

Load(sp) \longleftarrow *For each page in S*

For ep in **Pages**(EB[h]), st in sp, et in ep:

If (et.Sid=st.Sid):

Output(et \bowtie st)

*Cost = (pages in E in S) * IO cost*

How Much Memory?

- **Phase 1**
 - Space to store **current page** read for partitioning
 - Store one buffer page for **each hash bucket**
- **Phase 2**
 - Store all pages from **one hash bucket**
 - Store **current page** from other table bucket
 - One **output buffer** page

How Many Buckets?

- **Constraint in Phase 1**
 - $1 + \text{Nr. Buckets} \leq \text{Memory}$
- **Constraint in Phase 2**
 - $2 + \text{Nr. Pages in Smaller Table} / \text{Nr. Buckets} \leq \text{Memory}$
- **Rule of thumb**
 - Want memory $>$ **$\text{Sqrt}(\text{Nr. Pages in Smaller Table})$**

Example

Property	Value
Enrollment Pages	1,000
Student Pages	100
Available Buffer	11
Hash Join Cost	$\text{Sqrt}(100) < 11$ Cost: $3 * (100 + 1,000)$

Details on Calculations

- Have enough buffer space to execute join as discussed
 - Rule of thumb: $\text{Sqrt}(100) = 10 < 11$
- Phase 1 reads and writes each input table page once
 - Cost is $2 * (100 + 1,000)$
- Phase 2 reads and writes each input table page once
 - However, we do not count the output cost, as usual
 - Therefore, we only count cost $1 * (100 + 1,000)$

What If We Lack Memory?

- Number of buffer pages limits number of **output buckets**
- Not enough buckets means **too much data** per bucket
- Prevents us from **loading one bucket** entirely in Phase 2
- Hence, perform **multiple passes** over data in phase 1
 - In each pass, buckets are partitioned into **sub-buckets**
 - Iterate until data per bucket **fits** into main memory

Sort-Merge Join: Idea

- Also specific to **equality** join conditions
- **Phase 1 (Sort)**
 - **Sort** joined tables on the join column
- **Phase 2 (Merge)**
 - Efficiently **merge** sorted tables together

Join Phase 1: Overview

- Lots of **sorting algorithms** proposed in the literature
- However, typically assume that we access **single** entries
- But random data access can be very **inefficient**
- Hence, want to access **pages** of entries instead
- Need specialized ("**external**") sort algorithms

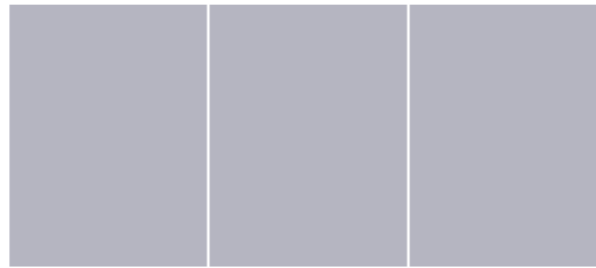
Algorithm Sketch

- **Step 1**: load chunk of data and **sort**, write back to disk
- **Step 2 .. n**: **merge** sorted runs to produce larger runs
- Each merging step **reduces** number of runs (but longer)
- Finally, have only **one sorted run** left - we're done!

Details on Step 1

- Assume we have **B buffer pages** available
- **Load chunks** of B pages into the buffer
- For each chunk, **sort** by standard sort algorithm
 - Can use standard algorithm as **all data in memory**
- Then, **write** sorted data to hard disk
- A sorted sequence of data is called a "**run**"

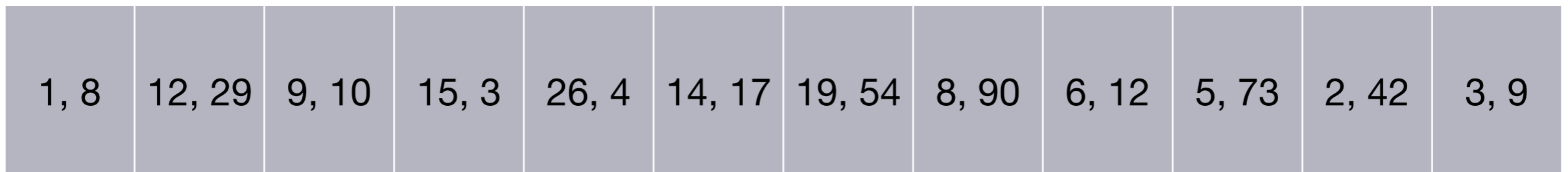
Step 1 Example



Buffer Pool (3 Pages)



Hard Disk (12 Pages)



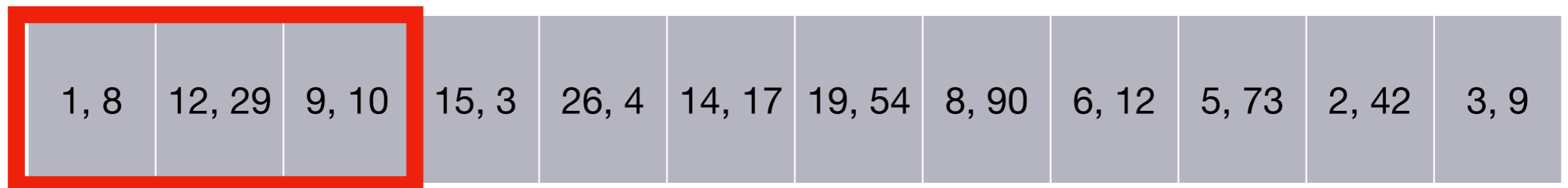
Step 1 Example



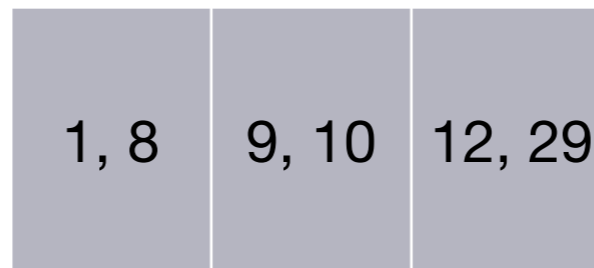
Buffer Pool (3 Pages)



Hard Disk (12 Pages)



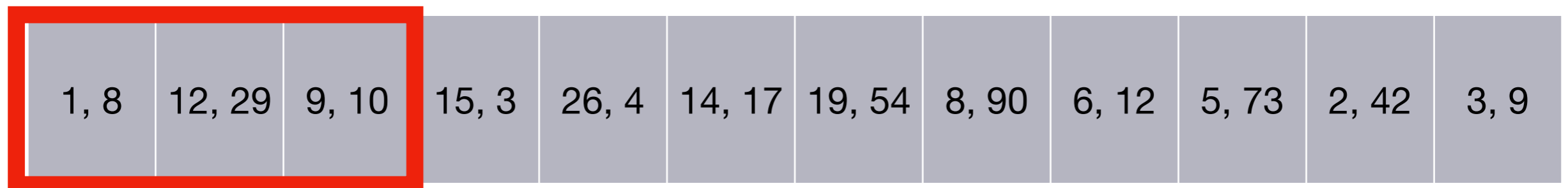
Step 1 Example



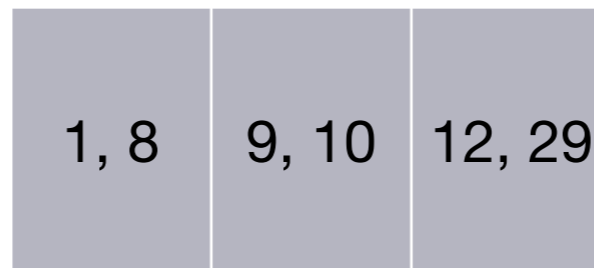
Buffer Pool (3 Pages)



Hard Disk (12 Pages)



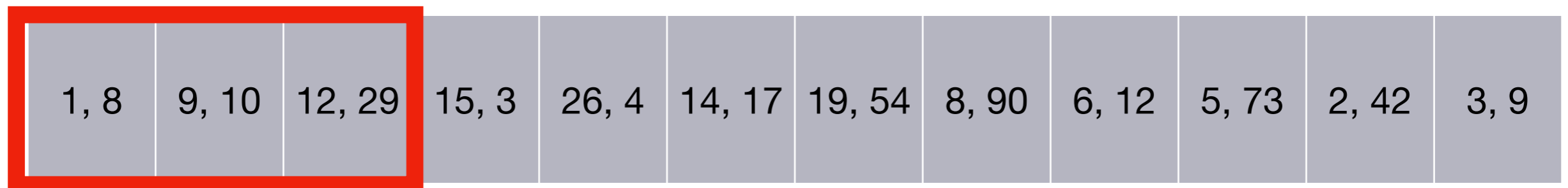
Step 1 Example



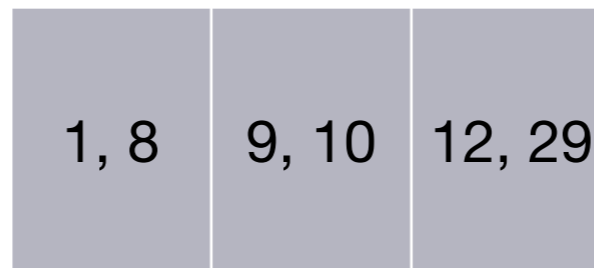
Buffer Pool (3 Pages)



Hard Disk (12 Pages)



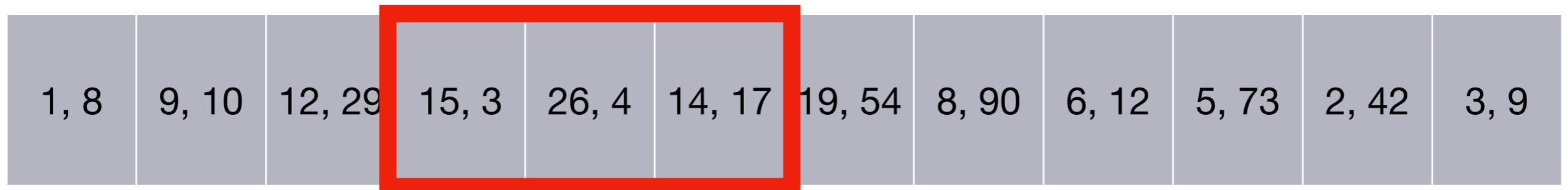
Step 1 Example



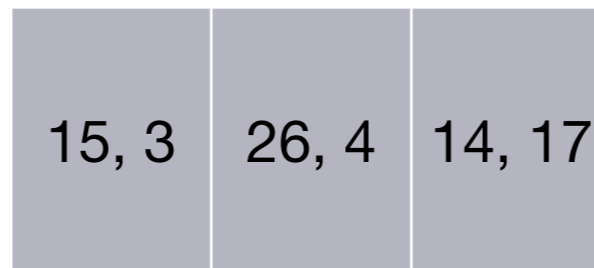
Buffer Pool (3 Pages)



Hard Disk (12 Pages)



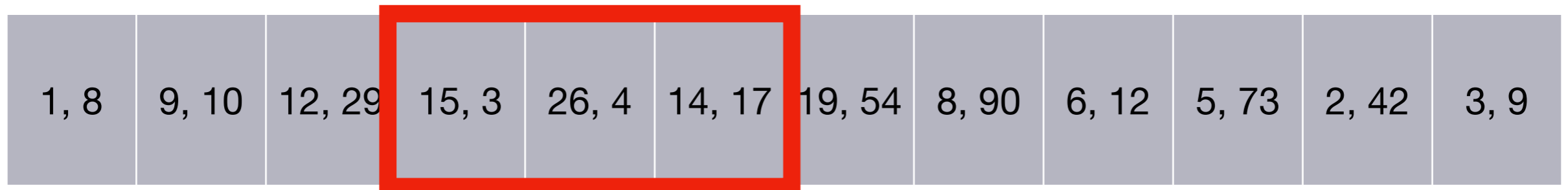
Step 1 Example



Buffer Pool (3 Pages)



Hard Disk (12 Pages)



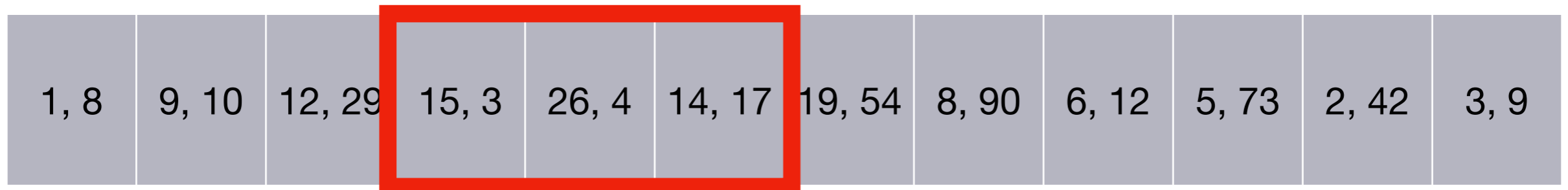
Step 1 Example



Buffer Pool (3 Pages)



Hard Disk (12 Pages)



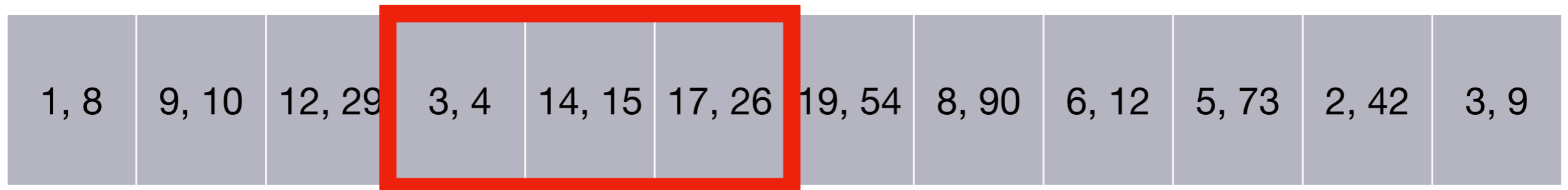
Step 1 Example



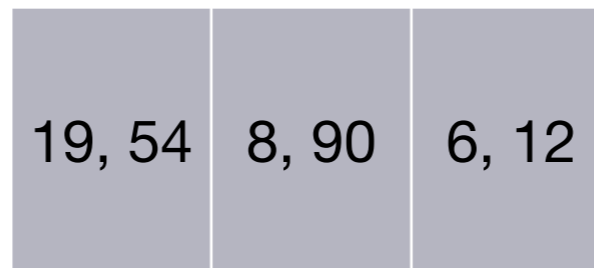
Buffer Pool (3 Pages)



Hard Disk (12 Pages)



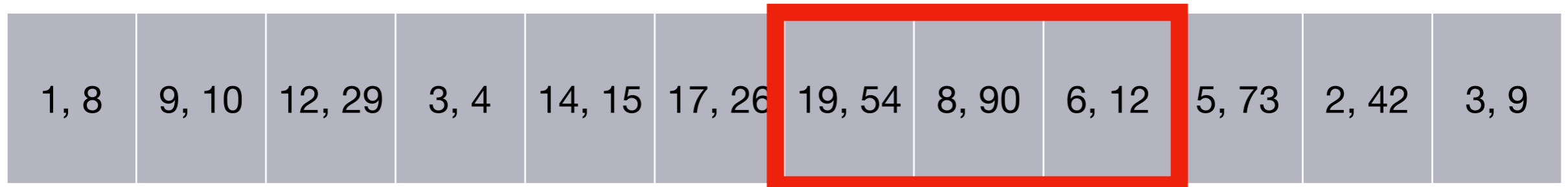
Step 1 Example



Buffer Pool (3 Pages)



Hard Disk (12 Pages)



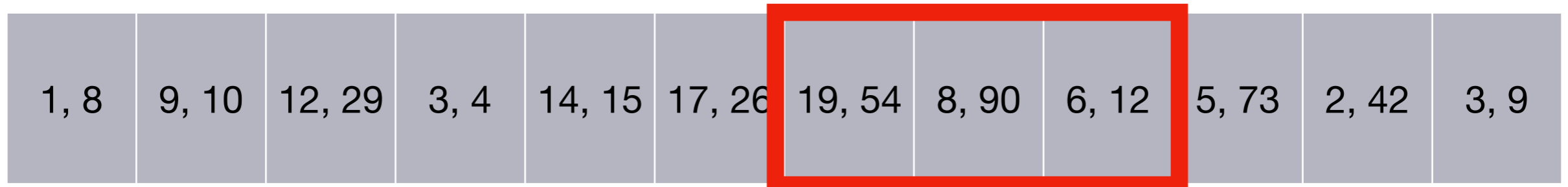
Step 1 Example



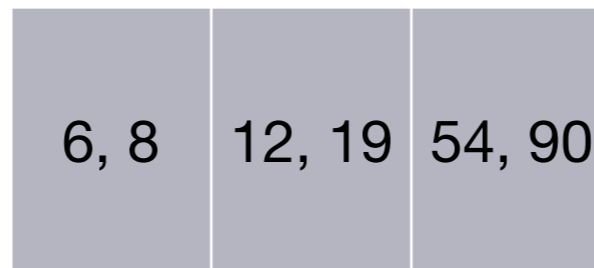
Buffer Pool (3 Pages)



Hard Disk (12 Pages)



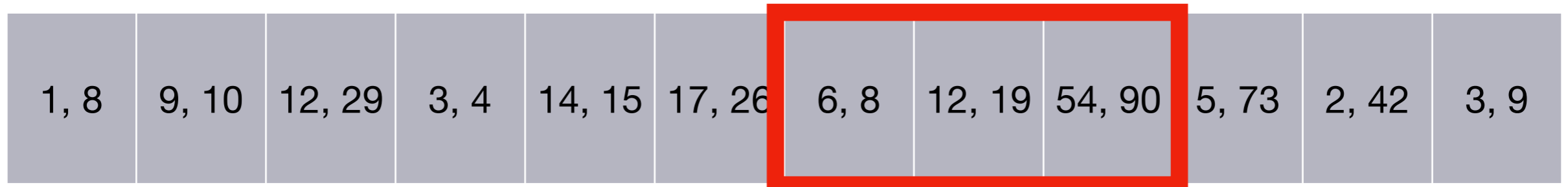
Step 1 Example



Buffer Pool (3 Pages)



Hard Disk (12 Pages)



Step 1 Example

6, 8	12, 19	54, 90
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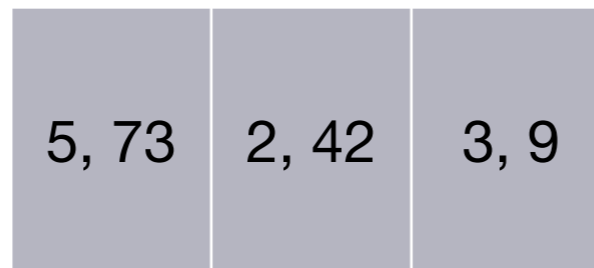
Buffer Pool (3 Pages)



Hard Disk (12 Pages)

1, 8	9, 10	12, 29	3, 4	14, 15	17, 26	6, 8	12, 19	54, 90	5, 73	2, 42	3, 9
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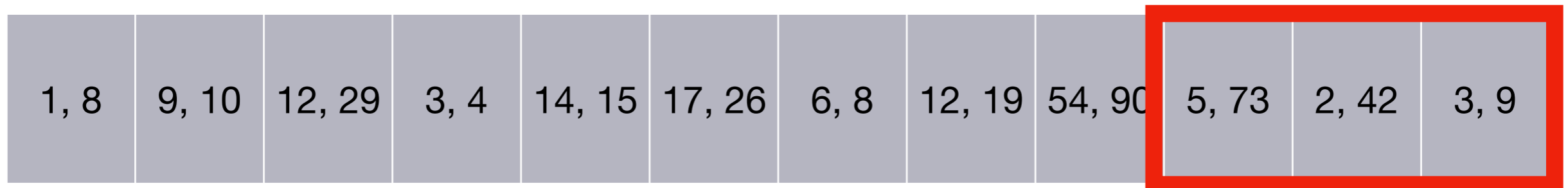
Step 1 Example



Buffer Pool (3 Pages)



Hard Disk (12 Pages)



Step 1 Example

2, 3	5, 9	42, 73
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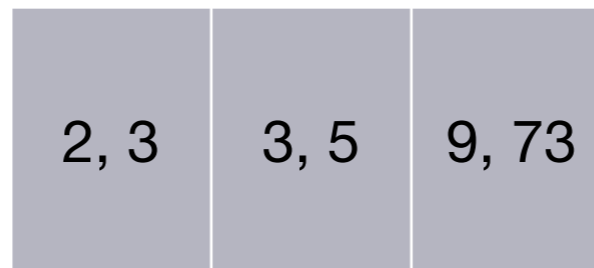
Buffer Pool (3 Pages)



Hard Disk (12 Pages)

1, 8	9, 10	12, 29	3, 4	14, 15	17, 26	6, 8	12, 19	54, 90	5, 73	2, 42	3, 9
------	-------	--------	------	--------	--------	------	--------	--------	-------	-------	------

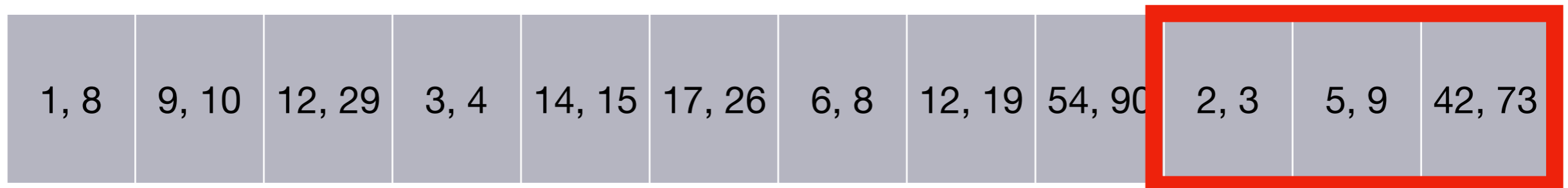
Step 1 Example



Buffer Pool (3 Pages)



Hard Disk (12 Pages)



Details on Steps 2 .. n

- (Still have B buffer pages available)
- Enables us to **merge $B-1$ sorted runs** into one in one step
 - **Load first page** of each sorted run into $B-1$ pages
 - **Copy minimum entry** in input buffers to output buffer
 - If output buffer full, **write to disk and clear**
 - **Erase minimum** entry from input buffer
 - If input buffer becomes empty, **load next page**

Step 2 Example

Input 1 *Input 2* *Output*



Buffer Pool (3 Pages)

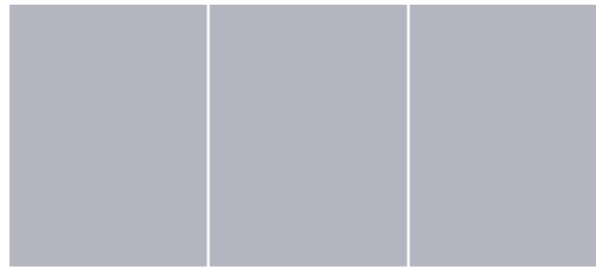


Hard Disk

1, 8	9, 10	12, 29	3, 4	14, 15	17, 26	6, 8	12, 19	54, 90	2, 3	5, 9	42, 73
------	-------	--------	------	--------	--------	------	--------	--------	------	------	--------

Step 2 Example

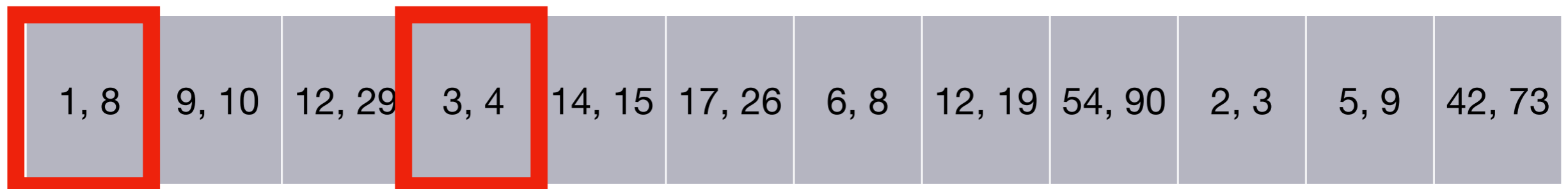
Input 1 *Input 2* *Output*



Buffer Pool (3 Pages)



Hard Disk



Step 2 Example

Input 1 *Input 2* *Output*

1, 8	3, 4	
------	------	--

Buffer Pool (3 Pages)

Hard Disk

1, 8	9, 10	12, 29	3, 4	14, 15	17, 26	6, 8	12, 19	54, 90	2, 3	5, 9	42, 73
------	-------	--------	------	--------	--------	------	--------	--------	------	------	--------

Step 2 Example

Input 1 *Input 2* *Output*

8	3, 4	1
---	------	---

Buffer Pool (3 Pages)

Hard Disk

1, 8	9, 10	12, 29	3, 4	14, 15	17, 26	6, 8	12, 19	54, 90	2, 3	5, 9	42, 73
------	-------	--------	------	--------	--------	------	--------	--------	------	------	--------

Step 2 Example

Input 1 *Input 2* *Output*

8	4	1, 3
---	---	------

Buffer Pool (3 Pages)

Hard Disk

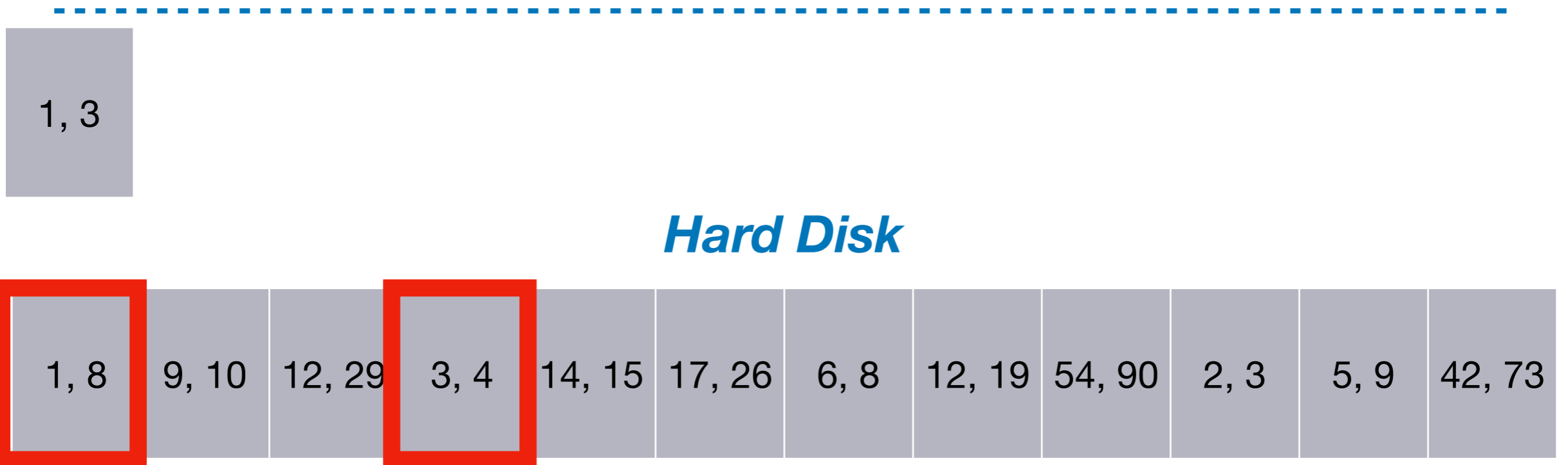
1, 8	9, 10	12, 29	3, 4	14, 15	17, 26	6, 8	12, 19	54, 90	2, 3	5, 9	42, 73
------	-------	--------	------	--------	--------	------	--------	--------	------	------	--------

Step 2 Example

Input 1 *Input 2* *Output*

8	4	
---	---	--

Buffer Pool (3 Pages)



Step 2 Example

Input 1 *Input 2* *Output*

8		4
---	--	---

Buffer Pool (3 Pages)

1, 3

Hard Disk

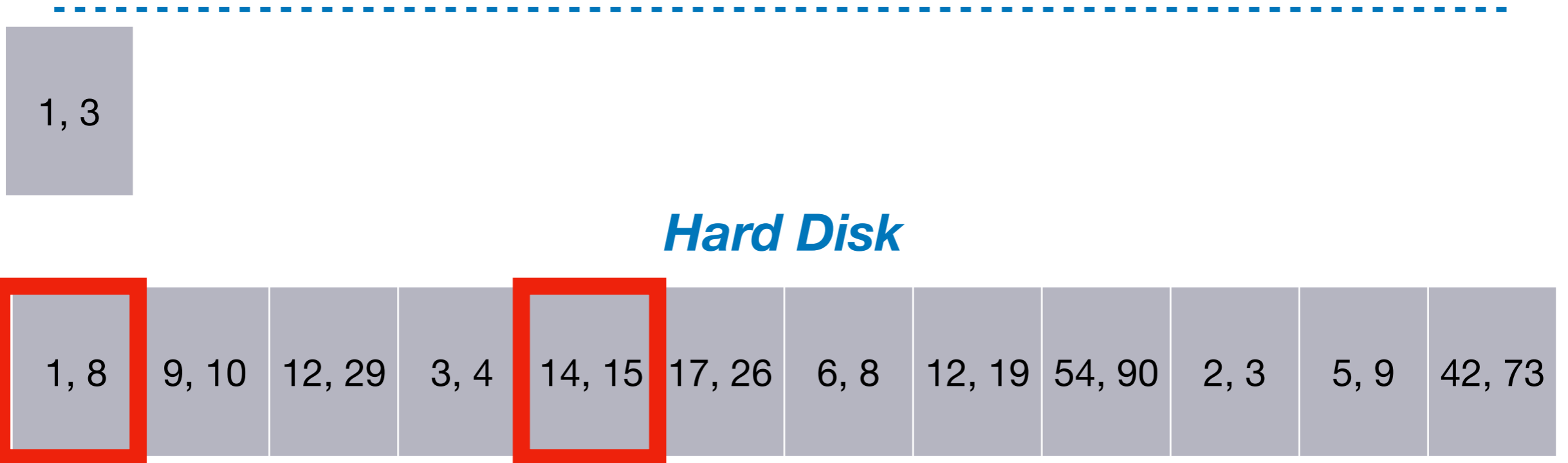
1, 8	9, 10	12, 29	3, 4	14, 15	17, 26	6, 8	12, 19	54, 90	2, 3	5, 9	42, 73
------	-------	--------	------	--------	--------	------	--------	--------	------	------	--------

Step 2 Example

Input 1 *Input 2* *Output*

8	14, 15	4
---	--------	---

Buffer Pool (3 Pages)

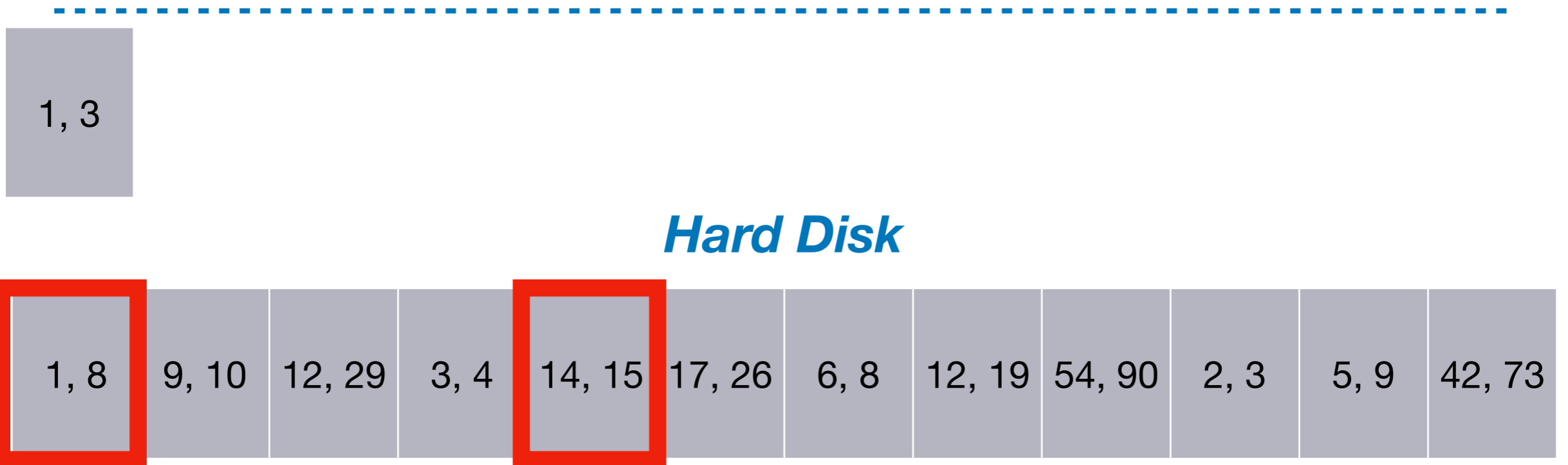


Step 2 Example

Input 1 *Input 2* *Output*

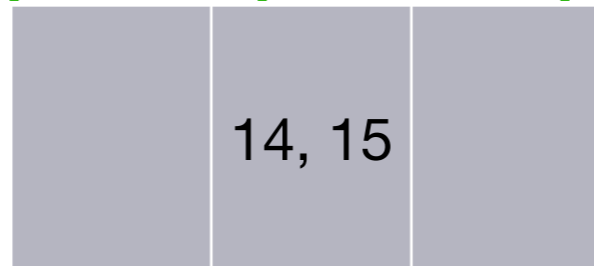
	14, 15	4, 8
--	--------	------

Buffer Pool (3 Pages)



Step 2 Example

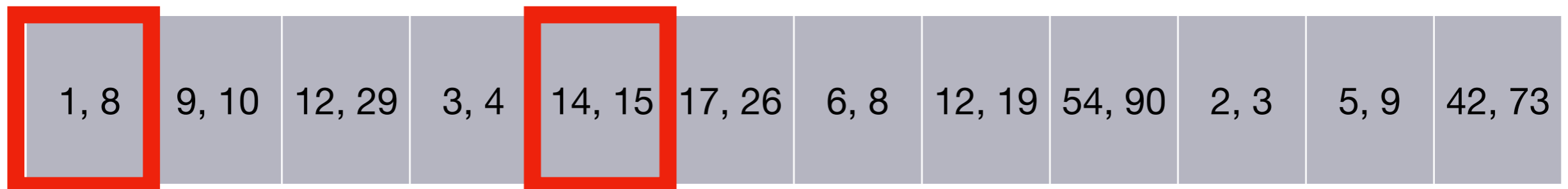
Input 1 *Input 2* *Output*



Buffer Pool (3 Pages)



Hard Disk



Step 2 Example

Input 1 *Input 2* *Output*

9, 10	14, 15	
-------	--------	--

Buffer Pool (3 Pages)

1, 3	4, 8
------	------

Hard Disk

1, 8	9, 10	12, 29	3, 4	14, 15	17, 26	6, 8	12, 19	54, 90	2, 3	5, 9	42, 73
------	-------	--------	------	--------	--------	------	--------	--------	------	------	--------

Step 2 Example

Input 1 *Input 2* *Output*

10	14, 15	9
----	--------	---

Buffer Pool (3 Pages)

1, 3	4, 8
------	------

Hard Disk

1, 8	9, 10	12, 29	3, 4	14, 15	17, 26	6, 8	12, 19	54, 90	2, 3	5, 9	42, 73
------	-------	--------	------	--------	--------	------	--------	--------	------	------	--------

Step 2 Example

Input 1 *Input 2* *Output*

	14, 15	9, 10
--	--------	-------

Buffer Pool (3 Pages)

1, 3	4, 8
------	------

Hard Disk

1, 8	9, 10	12, 29	3, 4	14, 15	17, 26	6, 8	12, 19	54, 90	2, 3	5, 9	42, 73
------	-------	--------	------	--------	--------	------	--------	--------	------	------	--------

Step 2 Example

Input 1 *Input 2* *Output*

12, 29	14, 15	
--------	--------	--

Buffer Pool (3 Pages)

1, 3	4, 8	9, 10
------	------	-------

Hard Disk

1, 8	9, 10	12, 29	3, 4	14, 15	17, 26	6, 8	12, 19	54, 90	2, 3	5, 9	42, 73
------	-------	--------	------	--------	--------	------	--------	--------	------	------	--------

Step 2 Example

Input 1 *Input 2* *Output*

29	14, 15	12
----	--------	----

Buffer Pool (3 Pages)

1, 3	4, 8	9, 10
------	------	-------

Hard Disk

1, 8	9, 10	12, 29	3, 4	14, 15	17, 26	6, 8	12, 19	54, 90	2, 3	5, 9	42, 73
------	-------	--------	------	--------	--------	------	--------	--------	------	------	--------



Example Summary

- Have **12 pages** to sort with **3 buffer** pages
- First step: produce **4 sorted runs** of **length 3**
- Can **merge 2 runs** in each merge step
- Second step: produce **2 sorted runs** of **length 6**
- Third step: produce **1 sorted run** of **length 12**

Cost Analysis (Phase 1)

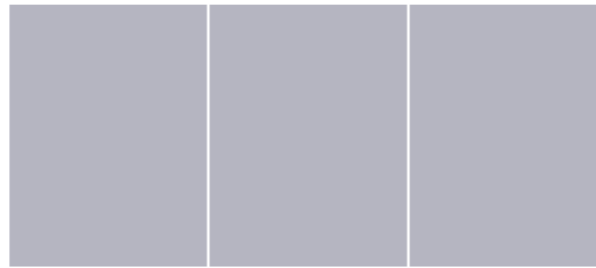
- Multiple **sorting passes**, we read and write data once in each
 - Cost per pass is **$2 * N$** (**N is number of pages**)
- **How many steps** must we make with B buffer pages?
 - **First step** produces runs of length B
 - **Second step** produces runs of length $(B-1) * B$
 - **Third step** produces runs of length $(B-1) * (B-1) * B \dots$
 - Stop once **$(B-1)^{\text{steps}-1} * B \geq N$** , after **$1 + \text{Ceil}(\log_{B-1}(N/B))$** steps

Join Phase 2: Overview

- (Have **sorted** both input tables by their join column)
- **Load first page** of both sorted tables into memory
- **Find matching tuples** and add to join result output
- **Load next page** for table with smallest last entry
- **Keep doing** until no pages left for one table

Join Phase 2 Example

Input 1 *Input 2* *Output*



Buffer Pool (3 Pages)



Hard Disk

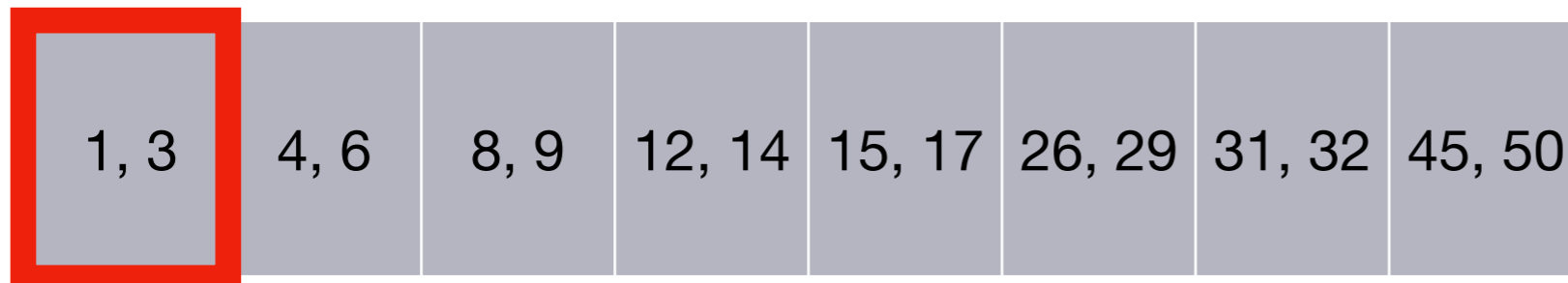


Table 1

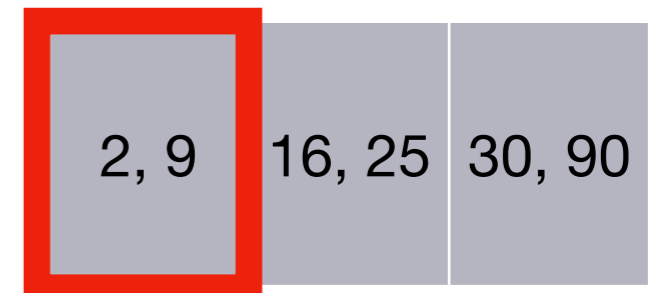


Table 2

Join Phase 2 Example

Input 1 Input 2 Output

1, 3	2, 9	
------	------	--

Buffer Pool (3 Pages)

Hard Disk

1, 3	4, 6	8, 9	12, 14	15, 17	26, 29	31, 32	45, 50
------	------	------	--------	--------	--------	--------	--------

Table 1

2, 9	16, 25	30, 90
------	--------	--------

Table 2

Join Phase 2 Example

Input 1 *Input 2* *Output*

3	2, 9	
---	------	--

Buffer Pool (3 Pages)

Hard Disk

1, 3	4, 6	8, 9	12, 14	15, 17	26, 29	31, 32	45, 50
------	------	------	--------	--------	--------	--------	--------

Table 1

2, 9	16, 25	30, 90
------	--------	--------

Table 2

Join Phase 2 Example

Input 1 *Input 2* *Output*

3	9	
---	---	--

Buffer Pool (3 Pages)

Hard Disk

1, 3	4, 6	8, 9	12, 14	15, 17	26, 29	31, 32	45, 50
------	------	------	--------	--------	--------	--------	--------

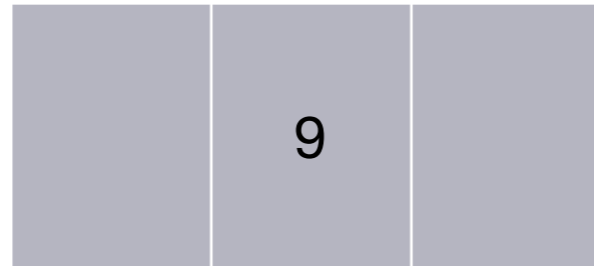
Table 1

2, 9	16, 25	30, 90
------	--------	--------

Table 2

Join Phase 2 Example

Input 1 *Input 2* *Output*



Buffer Pool (3 Pages)

Hard Disk

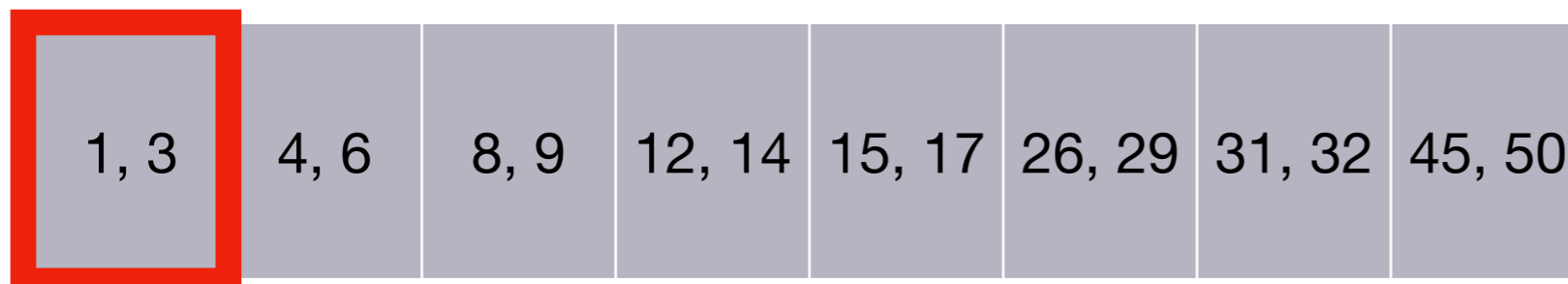


Table 1

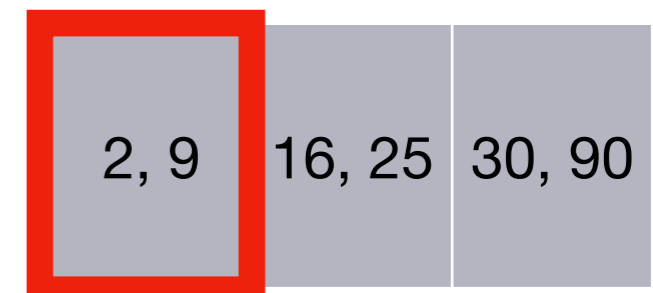


Table 2

Join Phase 2 Example

Input 1 *Input 2* *Output*

4, 6	9	
------	---	--

Buffer Pool (3 Pages)

Hard Disk

1, 3	4, 6	8, 9	12, 14	15, 17	26, 29	31, 32	45, 50
------	------	------	--------	--------	--------	--------	--------

Table 1

2, 9	16, 25	30, 90
------	--------	--------

Table 2

Join Phase 2 Example

Input 1 *Input 2* *Output*

6	9	
---	---	--

Buffer Pool (3 Pages)

Hard Disk

1, 3	4, 6	8, 9	12, 14	15, 17	26, 29	31, 32	45, 50
------	------	------	--------	--------	--------	--------	--------

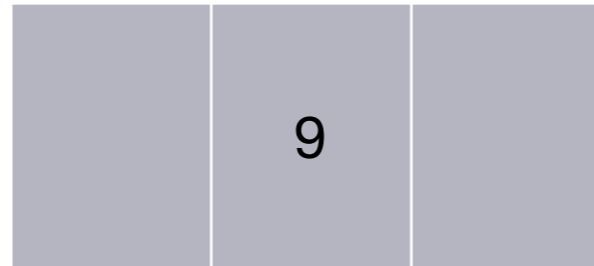
Table 1

2, 9	16, 25	30, 90
------	--------	--------

Table 2

Join Phase 2 Example

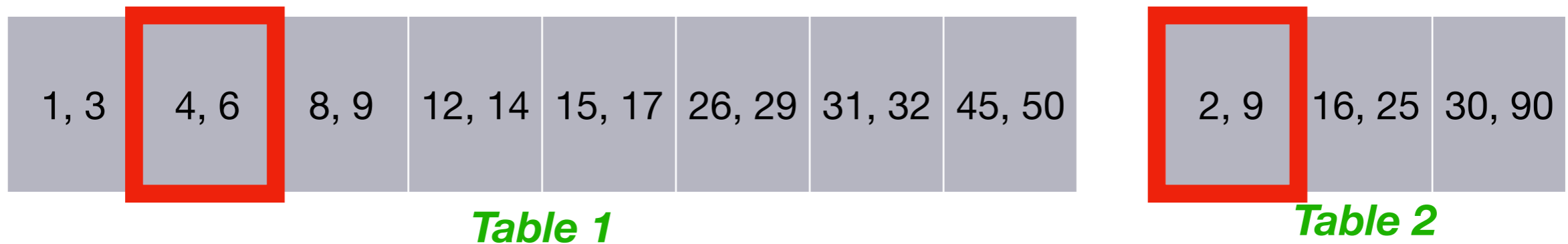
Input 1 *Input 2* *Output*



Buffer Pool (3 Pages)



Hard Disk



Join Phase 2 Example

Input 1 *Input 2* *Output*

8, 9	9	
------	---	--

Buffer Pool (3 Pages)



Hard Disk

1, 3	4, 6	8, 9	12, 14	15, 17	26, 29	31, 32	45, 50	2, 9	16, 25	30, 90
------	------	------	--------	--------	--------	--------	--------	------	--------	--------

Table 1

Table 2

Join Phase 2 Example

Input 1 *Input 2* *Output*

9	9	
---	---	--

Buffer Pool (3 Pages)



Hard Disk

1, 3	4, 6	8, 9	12, 14	15, 17	26, 29	31, 32	45, 50	2, 9	16, 25	30, 90
------	------	------	--------	--------	--------	--------	--------	------	--------	--------

Table 1

Table 2

Join Phase 2 Example

Input 1 *Input 2* *Output*

		9
--	--	---

Buffer Pool (3 Pages)



Hard Disk

1, 3	4, 6	8, 9	12, 14	15, 17	26, 29	31, 32	45, 50	2, 9	16, 25	30, 90
------	------	------	--------	--------	--------	--------	--------	------	--------	--------

Table 1

Table 2

Join Phase 2 Example

Input 1 *Input 2* *Output*

12, 14	16, 25	9
--------	--------	---

Buffer Pool (3 Pages)

Hard Disk

1, 3	4, 6	8, 9	12, 14	15, 17	26, 29	31, 32	45, 50
------	------	------	--------	--------	--------	--------	--------

Table 1

2, 9	16, 25	30, 90
------	--------	--------

Table 2

Join Phase 2 Example

Input 1 *Input 2* *Output*

14	16, 25	9
----	--------	---

Buffer Pool (3 Pages)

Hard Disk

1, 3	4, 6	8, 9	12, 14	15, 17	26, 29	31, 32	45, 50
------	------	------	--------	--------	--------	--------	--------

Table 1

2, 9	16, 25	30, 90
------	--------	--------

Table 2

Join Phase 2 Example

Input 1 *Input 2* *Output*

	16, 25	9
--	--------	---

Buffer Pool (3 Pages)

Hard Disk

1, 3	4, 6	8, 9	12, 14	15, 17	26, 29	31, 32	45, 50
------	------	------	--------	--------	--------	--------	--------

Table 1

2, 9	16, 25	30, 90
------	--------	--------

Table 2

Join Phase 2 Example

Input 1 *Input 2* *Output*

15, 17	16, 25	9
--------	--------	---

Buffer Pool (3 Pages)

Hard Disk

1, 3	4, 6	8, 9	12, 14	15, 17	26, 29	31, 32	45, 50
------	------	------	--------	--------	--------	--------	--------

Table 1

2, 9	16, 25	30, 90
------	--------	--------

Table 2

Join Phase 2 Example

Input 1 *Input 2* *Output*

17	16, 25	9
----	--------	---

Buffer Pool (3 Pages)

Hard Disk

1, 3	4, 6	8, 9	12, 14	15, 17	26, 29	31, 32	45, 50
------	------	------	--------	--------	--------	--------	--------

Table 1

2, 9	16, 25	30, 90
------	--------	--------

Table 2

Join Phase 2 Example

Input 1 *Input 2* *Output*

17	25	9
----	----	---

Buffer Pool (3 Pages)

Hard Disk

1, 3	4, 6	8, 9	12, 14	15, 17	26, 29	31, 32	45, 50
------	------	------	--------	--------	--------	--------	--------

Table 1

2, 9	16, 25	30, 90
------	--------	--------

Table 2

Join Phase 2 Example

Input 1 *Input 2* *Output*

	25	9
--	----	---

Buffer Pool (3 Pages)



Hard Disk

1, 3	4, 6	8, 9	12, 14	15, 17	26, 29	31, 32	45, 50	2, 9	16, 25	30, 90
------	------	------	--------	--------	--------	--------	--------	------	--------	--------

Table 1

Table 2

Join Phase 2 Example

Input 1 *Input 2* *Output*

26, 29	25	9
--------	----	---

Buffer Pool (3 Pages)

Hard Disk

1, 3	4, 6	8, 9	12, 14	15, 17	26, 29	31, 32	45, 50
------	------	------	--------	--------	--------	--------	--------

Table 1

2, 9	16, 25	30, 90
------	--------	--------

Table 2

Join Phase 2 Example

Input 1 *Input 2* *Output*

26, 29		9
--------	--	---

Buffer Pool (3 Pages)

Hard Disk

1, 3	4, 6	8, 9	12, 14	15, 17	26, 29	31, 32	45, 50
------	------	------	--------	--------	--------	--------	--------

Table 1

2, 9	16, 25	30, 90
------	--------	--------

Table 2

Join Phase 2 Example

Input 1 *Input 2* *Output*

26, 29	30, 90	9
--------	--------	---

Buffer Pool (3 Pages)

Hard Disk

1, 3	4, 6	8, 9	12, 14	15, 17	26, 29	31, 32	45, 50
------	------	------	--------	--------	--------	--------	--------

Table 1

2, 9	16, 25	30, 90
------	--------	--------

Table 2

Join Phase 2 Example

Input 1 *Input 2* *Output*

29	30, 90	9
----	--------	---

Buffer Pool (3 Pages)

Hard Disk

1, 3	4, 6	8, 9	12, 14	15, 17	26, 29	31, 32	45, 50
------	------	------	--------	--------	--------	--------	--------

Table 1

2, 9	16, 25	30, 90
------	--------	--------

Table 2



Handling Many Duplicates

- May have duplicates over **multiple pages**
- **Must revert** to first page with duplicate whenever we load new page from other table
- This makes the join more **expensive**

Cost Analysis (Phase 2)

- For now: assume that all **duplicate entries** on same page
 - Duplicate entry: **same value in join column**
- Means that each input page is **only read once**
- Cost is proportional to **number of input pages**
 - I.e., Pages from **both** input tables

Total Join Cost

- Two input tables with M and N pages, B buffer pages
- First phase has cost
 - $2 * M * (1 + \text{Ceil}(\log_{B-1}(M/B)))$ for sorting table 1
 - $2 * N * (1 + \text{Ceil}(\log_{B-1}(M/B)))$ for sorting table 2
- Second phase has cost
 - $M + N$ (we don't count cost for writing output!)

How Much Memory?

- First phase: try to exploit **all buffer pages**
 - More buffer means less merging passes!
- Second phase: only exploit **three buffer pages**
 - One for first input, one for second input, one output

How Much Memory?

- First phase: try to exploit **all buffer pages**
 - More buffer means less merging passes!
- Second phase: only exploit **three buffer pages**
 - One for first input, one for second input, one for output

Seems Sub-Optimal!

Refined Sort-Merge Join

- Idea: can merge **more than two** sorted tables in phase 2
- Hence, **do not need to sort** tables completely in phase 1
- Means we can **save steps** (i.e., passes over the data)
- **First phase**: only sort data chunks that fit into memory
- **Second phase**: join all sorted chunks together (one step)

Refined Join Details

- Assume B buffer pages, tables with N and M pages
- **First phase**: load chunks of B pages, sort, write back
 - We now have $(N+M)/B$ sorted chunks on disk
- **Second phase**: merge $B-1$ sorted chunks together
 - Can sort entries in-memory to find matches
- Cost is $2*(M+N)$ (Phase 1) + $1 * (M+N)$ (Phase 2)

How Much Memory?

- Again, B buffer pages, input sizes are M and N
- Have $(N+M)/B$ sorted runs after first phase
- Need $B-1 \geq (N+M)/B$ to merge them in one step
- Rule of thumb if $N > M$: need $B \geq 2*\text{Sqrt}(N)$

R-SMJ vs. Hash Join

	Hash Join	Refined Sort-Merge Join
Time	$3 * \text{Input Size}$	$3 * \text{Input Size}$
Memory	$> \text{Sqrt}(\text{Smaller Table Size})$	$> 2 * \text{Sqrt}(\text{Larger Table Size})$
Parallelization	Advantage	
Skew-Resistance		Advantage